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Pre-service Teacher Knowledge: Thinking About Conceptual Understanding

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ABSTRACT This paper reports on research conducted with pre-service teachers in a university in South Africa. Two groups of students, those who were doing a four-year teacher education degree and those who had completed a pure science and mathematics degree, were respondents in this research. The research reported in this paper is part of a larger study conducted. The data was analyzed using teacher knowledge, the Three Dimensions of Knowledge model and the commognitive process in mathematical thinking. The findings indicate that both groups lacked a deep conceptual understanding of gradients of straight lines, despite their being trained to practice as mathematics teachers the following year. Moreover, the data also showed that despite passing many university level mathematics modules, some of these pre-service teachers were uncertain about their understanding of the concept of gradient.

INTRODUCTION

In a study conducted by Mudaly and Moore-Russo (2011), it was found that the in-service teachers' understanding of the concept of gradient varied substantially. A large number of teachers in the study knew little or nothing about the concept. This paper reports on research that extended the 2011 study to look at how pre-service teachers understood the concept of gradient.

Many researchers have expressed the idea that South African learners perform very poorly in mathematics (McCarthy and Oliphant 2013; Alex and Mammen 2012; Taylor 2011). This study attempted to explore the pre-service teachers' understanding of the concept of gradient in order to establish whether problems at South African schools begin at teacher training institutions. Bleiler (2015) indicates that teacher education programs recognize the importance of teachers making connections between their mathematical knowledge and the pedagogy associated with it. However, this becomes problematic if the pre-service teachers do not know the mathematical knowledge themselves.

Teacher Knowledge

This study draws from the research egg model proposed by Ball et al. (2008). Their model differentiates between two broad categories of knowledge, namely, Subject Matter Knowledge and Pedagogical Content Knowledge. Subject Matter Knowledge (SMK) comprises of Common Content Knowledge (CCK), Horizon Content Knowledge (HCK) and Specialized Content Knowledge (SCK). CCK refers to the knowledge that teachers must develop in their learners. Teachers ought to know this knowledge well so that they will be able to answer learner questions and recognize errors that learners make. It is expected that mathematics teachers will be familiar with this type of knowledge. On the other hand, SCK is the knowledge all mathematics teachers ought to have. This knowledge is related to the actual teaching of mathematics. For example, it is essential for teachers to have sound knowledge of concepts that they teach as well as knowledge of different ways of representing these concepts. The model indicates that SCK is the most important of all the knowledge. In some ways, SCK may be viewed as the most important predictor of student performance. Ball et al. (2008) regard HCK as "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum". For example, teachers of Grade 11 classes must know what aspects of geometry were taught in all of the prior grades and what geometry will be taught in Grade 12. Essentially, this would mean that South African teachers who have a sound understanding of the Curriculum Assessment Policy Statements (CAPS) document across the grades would possess good horizon knowledge. These types of knowledge are directly related to the subject matter being taught.

Whilst knowledge of the content is important, Pedagogical Content Knowledge (PCK) is also essential. PCK comprises Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT) and Knowledge of Content and Curriculum (KCC). KCS refers to the knowledge that teachers ought to possess about how students learn particular mathematics content. This is often developed with experience and includes the teachers' knowledge of common misconceptions. KCT specifically refers to the knowledge of how different content ought to be taught. The teaching of fractions, for example, will be different from the teaching of triangles. This type of knowledge also includes the teachers' ability to design their classroom instruction and the way they choose and present examples using various representations. KCC is the knowledge that teachers possess on how the content fits into the curriculum and how choices are made about the sequential listing of topics and sections to be taught.

It is important to acknowledge that the each of the knowledge domains is not independent despite the knowledge egg placing great emphasis on SCK. There are overlaps and it is sometimes difficult to distinguish between the different knowledge domains, for example, between SCK and KCT.

Theoretical Framework

Mathematics teacher knowledge must also be viewed in conjunction with the three dimensions of knowledge, as proposed by Chaouki and Hasenbank in 2014. This model describes the intersecting influences of conceptual versus procedural knowledge, shallow versus deep or connected understanding, and novice versus practiced or experienced learners.

At the beginning of any section in mathematics, all learners begin as novices. This implies that their knowledge of concepts is isolated and disconnected from other related concepts and procedures will not be memorized and connected as well. However, as they develop as novices, they begin to form some connections with related concepts and procedures, although still not memorized, and begin to show some level of understanding. With greater practice, concepts become well memorized and procedures can be conducted by rote. Understanding is shallow at this stage. The final stage of the model indicates that with more practice, learners become better at the concepts being taught, and they slowly graduate towards a deeper understanding with various concepts becoming well understood and connected.

However, for a teacher of mathematics (one who must be considered to be highly *practiced*), all concepts must be well understood and well connected before teaching is facilitated. The teacher ought to have reached the state where concepts are thoroughly understood and connections between related concepts are strong. However, the lines between the conceptual and procedural knowledge could become blurred because the high level of understanding of the concept and the procedures involved remove the distinction between the concept and the procedure. This would then mean that in the light of the various domains of knowledge, pre-service teachers ought to have well connected and understood conceptual knowledge and be able to conduct procedures intelligently because they understand *why* it is done in a particular way.

Productive Thinking

In the attempt to assess the pre-service teachers' level of conceptual understanding, it is important to examine the way they think in terms of particular mathematical concepts. It may be conjectured that how a teacher thinks affects what he does in class, how s/he teaches, how s/he acts and what kind of experiences s/he may provide. For the teacher to become productive in class s/he needs to possess the various types of knowledge, become highly practiced in the subject (indicating well connected and well understood mathematical concepts) and s/he needs to demonstrate a level of thinking that is indicative of his/her understanding of these concepts.

In order to explore thinking, one needs to consider the broader discourse that mathematics teachers engage in. A discourse, in general, would refer to formal communication via oral or written means. Mathematics teachers engage in many sorts of discourses, with colleagues, learners and other professionals. This conveys the idea that discourses are external and involves communicating with others. Whether one is conscious of it or not, a duality exists about mathematical discourses (Sfard 2007). On the one hand is the idea that a discourse is external and deals with communicating via verbal or written means. This involves conveying ideas and knowledge to someone else. One often sees thinking as being separate from the discourse, the communicating of mathematical ideas. Thus, thinking is seen to be a process distinct from the discourse in which the mathematician engages. On the other hand, Sfard (2007) views this differently. She sees the act of thinking as part of a discourse as well. This she termed commognition as a term derived from the terms communication, which is an exchange of information and cognition, which is the process of acquiring knowledge. In commognition, thinking may be defined as the activity of communicating with oneself as new knowledge is constructed. Commognition assumes that thinking involves the process of exchanging information with oneself, using reasoning, intuition and perception. If a discourse is a formal communication, then thinking about mathematics and discovering mathematics are also discourses. Thinking can be viewed as communication with the self.

There are various types of discourses that a mathematics teacher ought to engage in, namely verbal and written discourses and now, according to Sfard (2007), thinking as a discourse. In particular, thinking involves communication related to reasoning, intuition and perception. Through reasoning, intuition and perception, teachers are expected to communicate that which they think to their learners using teacher talk and chalk. If the thinking is adequately visualized then it is possible that this can be communicated to others or to oneself in ways that are dependent on the level of understanding. It is conjectured that an ideal teacher would be one who displays good reasoning skills through visualization and can then communicate these concepts to him or herself and to the learners.

Objectives of this Research

This paper sought to answer two questions: Do pre-service teachers display a good conceptual understanding of the concept gradient for teaching in mathematics classrooms in schools in South Africa? How prepared are pre-service mathematics teachers to become practicing teachers in South African schools?

Zerpa et al. (2009) declare that the teachers' knowledge about teaching and learning ought to be recognized as the most important predictor of learner success. Even the advanced levels of teacher content knowledge may not be an accurate indication of learner success. Zerpa et al. (2009) further argue that there has been "*an implicit disagreement over the knowledge of mathematics that teachers need to know in order to teach with deep conceptual understanding*". This disagreement relates to whether the teachers' content knowledge is sufficient to produce deep conceptual understanding for learners or whether teachers need to be more proficient in pedagogical strategies related to the teaching of mathematics. To some extent, this debate should not be about which one is more important. It should really be about how one establishes a balance between the various types of knowledge a teacher ought to possess. Possessing different types of knowledge is essential for effective teaching to take place. This is further emphasized by Zerpa et al. (2009) when they state, "*teaching mathematics is a complex enterprise that entails making the content accessible, interpreting students' questions and ideas, and being able to explain concepts and procedures in different ways*". It is only through the acquisition of various types of knowledge that the teacher will be able to overcome the complexity of teaching.

MATERIAL AND METHODS

This qualitative research study is located in an interpretive philosophical paradigm, intended to establish the level of understanding of the concept of gradient and the level of preparedness exhibited by final year mathematics pre-service teachers in one tertiary institution in South Africa. These pre-service teachers had already completed five content modules and two method modules in mathematics. They had also completed a further sixteen modules, which included generic modules and modules in a second teaching major. All modules are intended to enhance the teaching of the pre-service teachers. The final year module had 79 registered pre-service teachers. Final year pre-service teachers were engaged in this exploration because it was assumed that they would be best prepared to teach, as they were to become fully qualified practicing teachers in the following year. Informed consent, including assuring participants about anonymity, autonomy and confidentiality, was obtained. They were informed that participation was voluntary. Sixty-nine of the students volunteered to participate and were given one and a half hours to complete the questionnaire. A further nine students, who were completing the Post Graduate Certificate in Education (PGCE), were also invited to participate. Seven of these PGCE students had completed a degree with mathematics, science and/or computer science as majors and two had completed a degree in commerce, which included a few mathematics courses. The questionnaire comprised 21 questions and varied in content. This paper reports on answers to selected questions and used commognition and the three dimensions of knowledge as a means of analyzing the data.

FINDINGS

The questionnaire was used to determine whether the pre-service teachers understood the concept of gradient and their level of preparedness for teaching in a mathematics class. After each question they were asked to explain their answers. In one of the questions, a number of graphs were presented and the respondents had to indicate whether the graphs had a gradient of 2. Further, as a means of establishing deep conceptual understanding, they were asked to indicate on the graph itself where they thought the gradient could be 2. This was mainly intended to elicit a response for the parabolas and circle graphs. The gradient of a straight line graph is amongst the most basic concepts that are taught at schools. Much of the subsequent work depends on this knowledge, including the work done in calculus at school and at university. It is therefore expected that those who have passed at least Grade 12 mathematics would have a fairly good understanding of the concept of gradient. This expectation increases for those who are majoring in mathematics with the intention of teaching it in schools. All pre-service teachers attempted a response to this question. Of the 78 pre-service teachers who responded to the question, 38 provided a valid response. Forty of the pre-service teachers were not certain. Their responses were either incomplete or incorrect. In general, those who did not provide a correct response implied that it could not be ascertained whether the graph could have a gradient of 2 because no other numerical values were provided. This was probably because the result of the fact that the intercepts were not indicated on the diagram. This in itself was problematic because the question did not require a calculation and therefore having intercept values was unnecessary. The pre-service teachers simply had to indicate whether the graph could have a gradient of 2 or not.

For the straight line graph (which had a positive gradient, with a negative y-intercept and a positive x-intercept), it was expected that the pre-service teachers would recognize that the slope of the graph implied an increase in the y-coordinate as the x-coordinate increased. This would then mean that the gradient was positive and hence there was a possibility that the gradient could have a value of 2. For those with a good conceptual understanding of the gradients of straight lines, the response would have been simple to deduce. If there was uncertainty, then the pre-service teacher would struggle to rationalize the possibility of the gradient being 2. Ten of the pre-service teachers who answered incorrectly stated that the gradient was negative. This implies that despite passing their matric examinations and at least five mathematics content modules at tertiary level, these pre-service teachers still did not possess a functional knowledge of gradients of a straight line. Usually, as novices, they would have memorized the connection between gradients and the shapes of the straight lines. Many of these pre-service teachers would themselves be teaching the same concept in the following year. They ought to have attained a high level of understanding and should have been able to identify the connections between the gradient, the shape, change in x and y values and the actual drawing of the graph.

Some pre-service teachers' understanding was so poor that they interpreted the negative y intercept as the determining factor for the sign of the gradient (they felt that due to a negative *y* intercept the gradient ought to be negative). The explanations provided for their answers also indicated that they possessed very little understanding of gradients of straight line graphs. One pre-service teacher stated that the gradient could not be 2 because "the line is pointed to the left". The graph has two arrows pointing in opposite directions and it is not exactly clear what this pre-service teacher understands in terms of the direction of the graph. The direction of the line ought to be considered with respect to an increasing x-value. This graph points in two directions but the angle that it makes with the x-axis is important to determine whether it has a positive or negative gradient. None of those pre-service teachers who provided the correct answer gave an explanation using the idea of the angle of the line with the x-axis. Perhaps it was not necessary because all they needed to do was determine whether the graph could have a gradient of 2.

Many of those who answered incorrectly also believed that this was because the calibration on the x and y axes were different. This too, ought not to have had a bearing on the answer because it is possible that the scale on the x and y axes could be different. Poor content knowledge was revealed when pre-service teachers reasoned that the gradient of the line was negative because the y-intercept was negative. The gradient of the graph is not solely determined by the y-intercept and whether the line cuts the y-axis above or below the x-axis should have little bearing on the sign of the gradient.

One pre-service teacher drew on the given graph, where the graph ought to be in order to have a gradient of 2 (refer to Fig. 1). An interesting aspect related to this response is the fact that she drew a graph that was parallel to the given graph and this would therefore mean that the gradient of his graph ought to be the same as that of the given graph. The only difference was that the new graph had a y-intercept that was positive. It seems that the pre-service teacher associated a positive gradient with a positive y-intercept, rather than the ratio of the y-intercept and x-intercept or rise over run or steepness. In a study conducted by Mudaly and Moore-Russo (2011), it was found that practicing teachers' understanding of gradient varied quite significantly. In fact, it was found that a number of teachers had no understanding of the concept of a gradient at all and many confused a positive gradient with a negative one.

There was also a level of uncertainty in the way the pre-service teachers in the current study answered. They lacked confidence and seemed to be unsure what the answer could be. Figure 2 illustrates an example of a pre-service teacher's response. The pre-service teacher first wrote

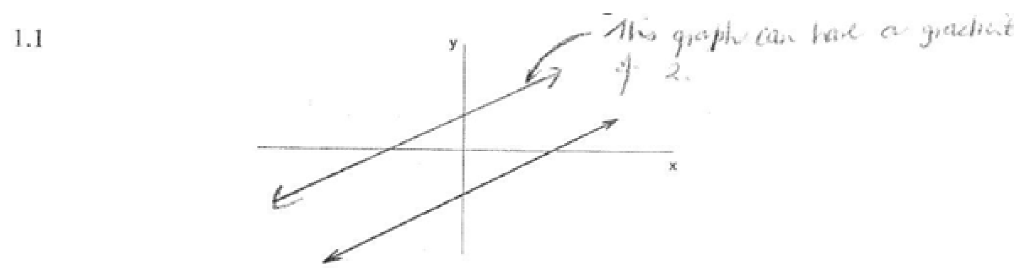

Fig. 1. Student response for possible gradient

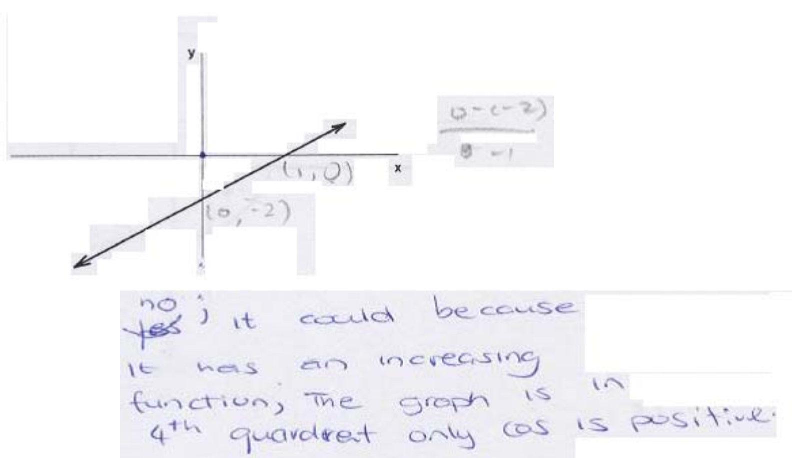

Fig. 2. Student's uncertain response

Yes, it could because it has an increasing function; the graph is in the 4[th] quadrant. Only cos is positive.

He was not sure and then struck off the 'yes' and replaced it with 'no'.

Another pre-service teacher (refer to Fig. 3) changed his mind in a similar way. The pre-service teacher began by indicating that the graph could have a gradient of 2 and then changed his mind mid-sentence. The response was:

Yes, the graph could have a...

He then struck off this response and wrote a response that was directly contrary to what he had initially indicated.

No, this graph could not have a gradient of 2. The equation of a straight line is not known and is unknown. If these are given, these must be in such a way that they will result in a gradient of 2.

This act of striking out an answer indicates that his thinking about the gradient was hazy and this might be attributed to a poor conceptual understanding of what a gradient actually is. He wrote as he was thinking and this commognitive process provides insights into how his discourse with himself was highly influenced by his poor conceptual understanding. If one has to rate the pre-service teacher using the three dimensions of knowledge tool, then this pre-service teacher is equivalent to a novice with a very shallow understanding of a gradient (Chaouki and Hasenbank 2014).

The response of a student, who had obtained a BSc degree, with mathematics as a major subject, demonstrates the commognitive processes that the student undergoes when faced with such a question (refer to Fig. 4). The student began with an indication of what his understanding of gradient was.

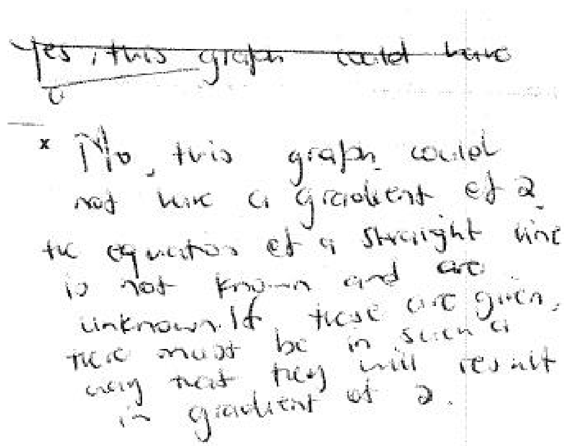

Fig. 3. A student response showing uncertainty

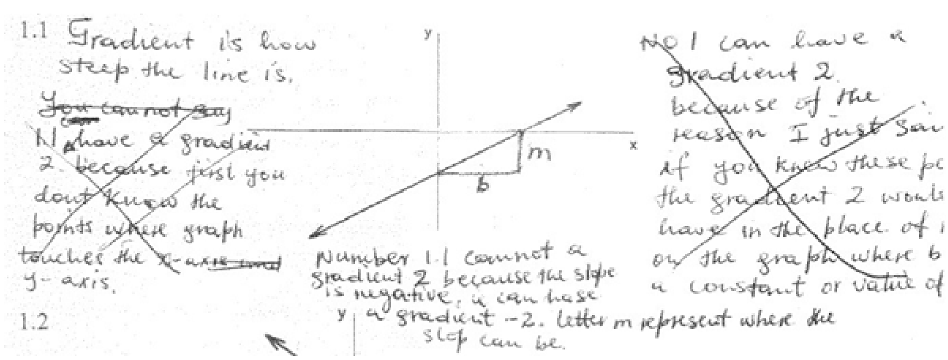

Fig. 4. A student response illustrating the commognitive process

The gradient is how steep a line is.

Perhaps this was a way of reaffirming his own understanding of a gradient, a way of talking to himself just to ensure that he could proceed with the solution. Using this first statement as a base, he decided that it was not possible to determine the gradient of the line because the intercepts were not provided.

You cannot say have a gradient 2 because first you don't know the points where the graph touches the x-axis and y-axis.

His argument hinges on the fact that there should be a numerical value attached to the intercepts and in the absence of these it would not be possible to find the gradient. His thinking about the problem did not cease. He then realizes that the graph could have a gradient of 2, but according to his way of thinking, this is only possible if you knew what the intercepts are. He struck off this response and then wrote:

No I can have a gradient of 2 because of the reason I just said. If you knew these points the gradient 2 would have in the place of m on the graph where b is a constant or value of y.

At this point he was still communicating with himself, but he encountered doubt. This doubt can possibly be attributed to his poor conceptual understanding of gradient. His communication with himself did not end there. He then struck out this statement as well and finally settled for a least expected response. His thinking eventually convinced him that it is not possible for the line to have a gradient of 2 because the slope of the line indicated that the gradient is negative.

Number 1.1 cannot (have) a gradient of 2 because the slope is negative that is, can have a gradient of -2.

It is evident that his thinking of gradients was seriously constrained by his poor conceptual understanding. This is despite the fact that he had completed his schooling and many undergraduate mathematics courses. In the context of the three dimensions of knowledge (Chaouki and Hasenbank 2014), this student will be in the region of a novice with disconnected and isolated understandings of the various concepts related to slope. The variation in his responses, as he attempted to find a solution, indicates that if there is no deep conceptual understanding then connections between different ideas become more difficult. For example, the student could not connect the physical shape of the graph to the idea that the y-value increases as the x-value increases. Neither could he see that the change in the y-value divided by the change in x-value will always be positive. Four attempts at finding the correct response are not ideal for any pre-service teacher. Finally, the student indicated that the gradient is represented by the letter *m* on the diagram (refer to Fig.3). This demonstrates that his understanding of gradients is poor because a gradient is not represented by a change in y values. It should be considered in terms of a ratio of the change in y divided by a change in x-values.

DISCUSSION

Conceptual Understanding

Orrill and Kittleson (2015) state that in order to establish a coherent understanding of mathematics, learners need to "form connections between and among concepts". This study showed that the pre-service teachers' poor understanding of the concept of gradient was linked to their inability to form links between the gradient, shape of graph, ratio of the changes in x and y-values, angle with the x-axis and the sign of the gradient value. This was demonstrated by the number of pre-service teachers who declared that the gradient of the given line is negative or by the fact that they drew just one line parallel to the given line. They ought to have known that parallel lines have the same gradient.

There seems to be a need for these pre-service teachers to relate the gradient to some numerical value and they could not imagine how a graph could have a gradient of two without the existence of actual numerical values. This idea was conveyed when some pre-service teachers stated that they could not establish whether the gradient could be two because no values for the intercepts were given. Furthermore, very few pre-service teachers actually recognized that an infinite number of other graphs could have been drawn with the gradient of the given graph. Many of these pre-service teachers also needed to see the calibration on the axis system. They could not link their understanding of the concept of the axis system to an abstract, hypothetical value of the gradient.

Knowledge of Mathematics

The empirical evidence suggests that the pre-service teachers showed deficiencies in Subject Matter Knowledge related to gradients. In particular, their Specialized Content Knowledge, which all mathematics teachers ought to know, was for many of these pre-service teachers, very limited. The study found that it did not matter whether these pre-service teachers completed specialized mathematics modules or not, the limitations related to their knowledge was still prevalent. The pre-service teachers who had graduated with a Bachelor of Science degree and whose mathematics modules had been at a higher level, presented similar responses to those pre-service teachers completing a Bachelor of Education degree, where the level of mathematics was distinctly lower.

With limited SCK, it is difficult to envisage how these pre-service teachers could become effective mathematics teachers. Poor SCK would influence their Common Content Knowledge and their Pedagogical Content Knowledge.

Conceptual Knowledge

Using the three dimensions of knowledge model (Chaouki and Hasenbank 2014), it would seem that the pre-service teachers' knowledge about gradients was shallow and disconnected. Ndlovu and Brijlall (2015) stated that in order for learners to develop conceptual understanding they need to form relationships with other related concepts. Some of the pre-service teachers' knowledge in this study showed little connection between the different concepts related to gradients. They for example, could not relate the direction of the graph to the sign of the gradient.

Their knowledge was generally isolated and in some instances, not well memorized. This was evident when they stated that the graph had a negative gradient. The empirical evidence suggests that some of these pre-service teachers remained at the level of a novice, despite the many years of study. In order for effective teaching to occur, these pre-service teachers should be at the level of an expert.

Bartell et al. (2013) emphasized the idea that pre-service teachers ought to develop conceptual knowledge when they stated, "teaching mathematics for conceptual understanding is a fundamental goal of mathematics teacher education". Further, Stohlmann et al. (2015) declare, "it is important that mathematics content courses for pre-service teachers are structured with a focus on conceptual understanding".

Mathematical Discourse

Many of these pre-service teachers showed uncertainty in their knowledge through their insecure responses. This was evident in the way they wrote out a response and then struck it off and replaced it with another response. In some instances, they showed multiple changes in their responses. Their thinking, which represented an intra-personal communication with themselves, indicated that their understanding of the concept of gradient was shallow. This would suggest that when they engage in inter-personal communication, with their students for example, they might find it difficult to ensure successful understanding of the concept. It may be assumed that their poor conceptual knowledge influenced the commognitive discourse that they engaged in.

Lloyd (2014), in similar research, also found that "prospective teachers will need to improve their proficiency with algebraic thinking in order to offer effective instruction to their students". It is useful, when attempting to ascertain a person's understanding of mathematical concept, to explore the way they think. Thinking is very difficult to measure or assess but one can look at how productive a person is when confronted with mathematical problems. This ought to be done by "recognizing that communication in mathematical learning is an aid to, or a component of thinking" (Nardi et al. 2014).

CONCLUSION

There is evidence that the pre-service teachers in this study, despite having completed many undergraduate mathematics courses, still display poor conceptual understanding of gradients of straight lines. This is observed through their responses to the particular questions analyzed in this paper. It seems that in terms of gradients of straight lines, these pre-service teachers still find themselves making little or no connections between the various related notions. The finding that needs to be highlighted is the idea that even those students who completed a pure mathematics degree demonstrate a poor conceptual understanding of gradients and this can be seen in the way they "think" about the gradients.

A further conclusion of this study is that these pre-service teachers may not be well prepared to begin teaching in mathematics classrooms, especially with respect to the concept of gradient. If their knowledge shows gaps in understanding then it may be reasonably assumed that their ability to teach these concepts effectively may be compromised. This may provide some reasons for the poor performance of South African students in international assessments.

RECOMMENDATIONS

There are some recommendations that this research may suggest. Firstly, teacher education institutions need build into their curriculum the idea of evaluating how students think when working with mathematics concepts. There should be more writing by pre-service teachers about their own thinking using journaling and diaries.

Secondly, teacher education institutions must find ways of guiding pre-service teachers to find ways of creating a coherent understanding of mathematical concepts and the way they link different but related concepts.

Thirdly, teacher education institutions need to evaluate their programs in a way that ensures that pre-service teachers who do not know the concepts themselves are not immediately sent out to teach at schools.

The fourth recommendation suggests that there are deficiencies in the way all mathematics modules are taught. These deficiencies need to be identified and modified so that pre-service teachers can exit the system with a reasonably good knowledge of mathematics for teaching.

Finally, university mathematics education staff should reflect on the way concepts are taught so that the pre-service teachers may be able to reflect on the way they then teach mathematics. Staff presentations must model the way a mathematician should 'think' when working with the different mathematical concepts.

LIMITATIONS

The only limitation of this study is that it was conducted with one class of final year Bachelor of Education and one class of Post Graduate certificate of Education pre-service teachers. Whilst this may be the case, the researcher is convinced that the findings would not have changed.

REFERENCES

Alex JK, Mammen KJ 2012. A survey of South African Grade 10 learners' geometric thinking levels in terms of the Van Hiele theory. *Anthropologist*, 14(2): 123-129.

Ball DL, Thames MH, Phelps G 2008. Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5): 1-55.

Bartell TG, Webel C, Bowen B, Dyson N 2013. Prospective teacher learning: Recognizing evidence of conceptual understanding. *Journal of Mathematics Teacher Education,* 16: 57–79.

Bleiler SK 2015. Increasing awareness of practice through interaction across communities: The lived experiences of a mathematician and mathematics teacher educator. *Journal of Math Teacher Education,* 18: 231–252.

Chaouki R, Hasenbank JF 2013. Improving Understanding in Algebra 1: A Deeper Understanding of Algebra/ Math. From <http://www.slideserve.com/elina/a-deeper-understanding-of-algebra-1188586>

Lloyd GM 2014. Research into teachers' knowledge and the development of mathematics classroom practices. *Journal of Mathematics Teacher Education,* 18: 393–395.

McCarthy J, Oliphant R 2013. *Mathematics Outcomes in South African Schools: What are the Facts? What Should be Done?* Johannesburg: Centre for Development and Enterprise.

Mudaly V, Moore-Russo D 2011. South African teachers' conceptualisations of gradient: A study of historically disadvantaged teachers in an Advanced Certificate in Education programme. *Pythagoras,* 32(1): 27-34.

Elena Nardi, Andreas Ryve, Erika Stadler, Olov Viirman 2014. Commognitive analyses of the learning and teaching of mathematics at university level: The case of discursive shifts in the study of Calculus. *Research in Mathematics Education,* 16(2): 182-198.

Ndlovu Z, Brijlall D 2015. Pre-service teachers' mental constructions of concepts in matrix algebra. *African Journal of Research in Mathematics, Science and Technology Education,* 19(2): 156-171.

Orrill CH, Kittleson JM 2015. Tracing professional development to practice: Connection making and content knowledge in one teacher's experience. *Journal of Mathematics Teacher Education,* 18: 273–297.

Sfard A 2007. When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. *The Journal of the Learning Sciences,* 16(4): 567–615.

Stohlmann M, Cramer K, Moore T, Maiorca, C 2015. Changing pre-service elementary teachers' beliefs about mathematical knowledge. *Mathematics Teacher Education and Development,* 16(2): 4-28

Taylor N 2011. *Priorities for Addressing South Africa's Education and Training Crisis: A Review Commissioned by the National Planning Commission*. Johannesburg: Jet Education Services.

Zerpa C, Kajander A, Van Barneveld C 2009. Factors that impact pre-service teachers' growth in conceptual mathematics knowledge during a mathematics methods course. *International Electronic Journal of Mathematics Education,* 4(2): 57-76.